

Magnetic Tuning of Cylindrical $H_{01\delta}$ -Mode Dielectric Resonators

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Abstract—The theory of magnetic tuning of cylindrical $H_{01\delta}$ dielectric resonators is developed. It is based on rigorous solutions to the dielectric resonator systems containing microwave ferrites. It is shown that the most effective magnetic tuning of $H_{01\delta}$ dielectric resonators can be accomplished by inserting a thin ferrite rod through an axial hole in the resonator. This kind of tuning utilizes the dependence of the parallel permeability tensor component μ_z on the magnetic field applied. Experiments have been performed which show that a 4 percent tuning range can be attained with a Q factor of the resonant system of the order of 2000 at X-band. Using an appropriate dc magnetic field circuit, a 120 MHz tuning bandwidth has been obtained with a consumption of tuning power of about 75 mW.

I. INTRODUCTION

THE RESONANT frequencies of dielectric resonators are determined by the resonator material permittivity, resonator dimensions, and the shielding conditions [1], [2]. The resonant frequency can be magnetically tuned by attaching a microwave ferrite to the resonator and applying a magnetic field to it. The magnetic field controls the permeability of the ferrite and hence the resonant frequency of the resonant system. Tuning bandwidths of the order of 1 percent [3] and 2 percent [4] have been reported. The authors of the above-mentioned papers chose ferrites in the form of disks attached above the dielectric resonator. Such configurations were chosen rather intuitively since a theory of tuning has not been developed to date. This paper presents systematic studies of ferrite tuning of cylindrical $H_{01\delta}$ -mode dielectric resonators. The theory is based on rigorous Rayleigh–Ritz solutions to the resonant systems containing ferrites. The geometries under study are illustrated schematically in Fig. 1. It is assumed that the external magnetic field is uniform and directed along the z axis of the cylindrical coordinate system, so the permeability tensor of the ferrite medium is characterized by the following expression:

$$\vec{\mu} = \begin{bmatrix} \mu & -j\kappa & 0 \\ j\kappa & \mu & 0 \\ 0 & 0 & \mu_z \end{bmatrix} \quad (1)$$

where $\mu = \mu' - j\mu''$, $\kappa = \kappa' - j\kappa''$, and $\mu_z = \mu'_z - j\mu''_z$. The relative permittivities of the ferrite medium and of the dielectric resonator are assumed to be scalars, denoted by $\epsilon_f = \epsilon'_f - j\epsilon''_f$ and $\epsilon = \epsilon' - j\epsilon''$, respectively.

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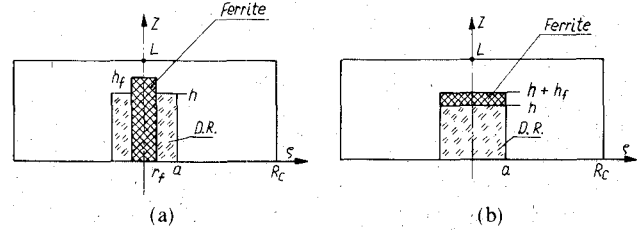


Fig. 1. Structures of shielded magnetically tunable dielectric resonators. (a) Cylindrical dielectric ring resonators with a gyromagnetic rod. (b) Cylindrical dielectric resonator with a gyromagnetic disk.

II. THEORY

The resonant frequencies of the resonant system shown in Fig. 1 can be found as the solutions to the following eigenvalue problem:

$$L\vec{\Phi} = j\omega M\vec{\Phi} \quad (2)$$

$$\vec{n} \times \vec{E} = 0 \quad \text{on } S$$

where

$$L = \begin{bmatrix} 0 & \text{curl} \\ \text{curl} & 0 \end{bmatrix} \quad M = \begin{bmatrix} \epsilon_0 \epsilon(r) & 0 \\ 0 & -\mu_0 \vec{\mu}(r) \end{bmatrix}$$

$$\vec{\Phi} = \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix}$$

Rigorous solutions to the problem given by (2) have been obtained by means of the Rayleigh–Ritz method [5], [6] using empty cavity functions as a basis [7]. The resonant frequencies have been found for the lossless system. The Q factors of the resonant system can be computed using perturbation theory [8], [2]. We shall remark, however, that for low-loss systems this theory leads to exact results. Following this theory, the Q factor of the resonant system with a perfectly conducting shield is computed as follows:

$$Q^{-1} = p_\epsilon \epsilon''/\epsilon' + p_{\epsilon_f} \epsilon''_f/\epsilon'_f + p_\mu \mu''/\mu' + p_\kappa \kappa''/\kappa' + p_{\mu_z} \mu''_z/\mu'_z \quad (3)$$

where $p_x = 2|\partial f_r/\partial x|/f_r$ is the electric or magnetic energy filling factor; x stands for ϵ' , ϵ'_f , μ' , κ' , or μ'_z respectively; and f_r is the resonant frequency.

All the differentials required for p_x factor calculations can be computed numerically with the aid of the program used for the resonant frequency computations.

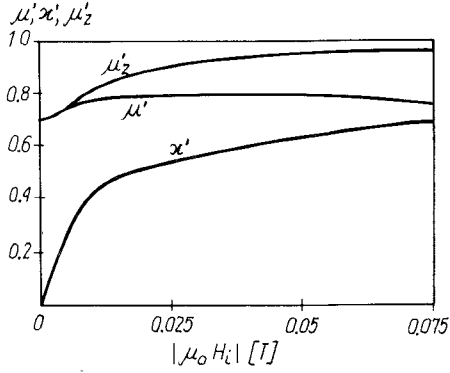


Fig. 2. Permeability tensor components versus internal dc magnetic field applied for the lithium-titanium ferrite used in experiments. Measurements were performed at a frequency of about 9300 MHz.

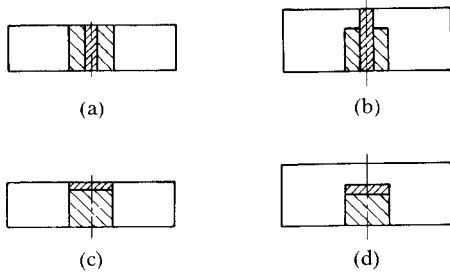


Fig. 3. Configurations of the resonant systems for the Rayleigh-Ritz analysis. (a) Shielded quasi- H_{011} -mode dielectric ring resonator with a ferrite rod. $R_c = 16$ mm, $a = 4$ mm, $r_f = 1.5$ mm, $h = L = 8$ mm, $\epsilon' = \epsilon_f' = 15$. (b) Shielded quasi- H_{018} -mode dielectric ring resonator with a ferrite rod. $R_c = 16$ mm, $a = 4$ mm, $r_f = 1.5$ mm, $h = 8$ mm, $L = 12$ mm, $\epsilon' = \epsilon_f' = 15$. (c) Shielded quasi- H_{018} -mode dielectric resonator with a ferrite disk. $R_c = 16$ mm, $a = 4$ mm, $h = 6.5$ mm, $h_f = 1.5$ mm, $L = 8$ mm, $\epsilon' = \epsilon_f' = 15$. (d) Shielded quasi- H_{018} -mode dielectric resonator with a ferrite disk. $R_c = 16$ mm, $a = 4$ mm, $h = 6.5$ mm, $h_f = 1.5$ mm, $L = 12$ mm, $\epsilon' = \epsilon_f' = 15$.

The tuning characteristics of a resonant system containing ferrites depend on ferrite properties. Since low tuning power is usually desirable, properties of partially magnetized ferrites are needed. In Fig. 2 are shown permeability tensor components versus the dc magnetic field intensity for the lithium-titanium ferrite (made by Polfer, Poland) used in our experiments. The characteristics shown in Fig. 2 are valid if the curves start from the completely demagnetized state of the ferrite. It will be shown in the experimental part of this paper that hysteresis effects exert an important influence on the tuning curves of the resonant system.

Now we consider the possibilities of tuning the different quasi- H_{018} -mode resonant systems shown in Fig. 3. The parameters of the systems were chosen so as to be similar to those used in our experiments. Computations have been performed using basis functions sketched schematically in Fig. 4. The meaning of particular basis functions is fully explained in [7]. The electric type basis functions must be included since generally $\kappa \neq 0$, so $E_z \neq 0$ (this explains the designation quasi- H_{018} mode, since for the H_{018} mode $E_z = 0$). The results of the computations of the quasi- H_{018} -mode resonant frequencies, the electric and the magnetic energy filling factors, are presented in Table I. One

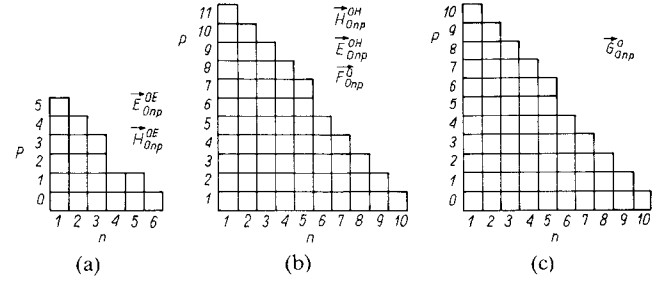


Fig. 4. Subscripts of basis functions used for computations of the resonant frequencies and the electric and magnetic energy filling factors of the resonant systems shown in Fig. 3. (a) Rotational electric type basis functions. (b) Rotational magnetic type basis functions and potential electric type basis functions. (c) Potential magnetic type basis functions.

TABLE I
QUASI- H_{011} -MODE RESONANT FREQUENCIES AND THE ELECTRIC AND MAGNETIC ENERGY FILLING FACTORS FOR THE RESONANT SYSTEMS SHOWN IN FIG. 3

Case (n Fig 3)		$\epsilon_f = 15$ $\mu' = \mu_z' = 1$ $\kappa = 0$	$\epsilon_f = 15$ $\mu' = \mu_z' = 0.7$ $\kappa = 0$	$\epsilon_f = 15$ $\mu' = \mu_z' = 0.7$ $\kappa = 0.5$	$\epsilon_f = 1$ $\mu' = \mu_z' = 1$ $\kappa = 0$
a)	f_r (MHz)	9729.6	10215.6	10210.5	9959.6
	P_E	0.9051	0.9223	—	—
	P_{E_f}	0.0565	0.0333	—	—
	P_{μ_z}	0.3004	0.2258	—	—
	P_{μ_x}	0.0139	0.0073	—	—
	P_{μ_y}	0.0023	0.0019	—	—
b)	f_r (MHz)	9019.6	9503.2	9502.3	9223.3
	P_E	0.8799	0.8919	—	—
	P_{E_f}	0.0530	0.0321	—	—
	P_{μ_z}	0.3292	0.2445	—	—
	P_{μ_x}	0.0085	0.0044	—	—
	P_{μ_y}	0.0002	0.0002	—	—
c)	f_r (MHz)	9729.55	9892.1	9811.0	9870.0
	P_E	0.9229	0.9377	—	—
	P_{E_f}	0.0393	0.0239	—	—
	P_{μ_z}	0.0237	0.0164	—	—
	P_{μ_x}	0.0798	0.0655	—	—
	P_{μ_y}	0.0266	0.0283	—	—
d)	f_r (MHz)	9029.8	9240.6	9237.4	9510.1
	P_E	0.7758	0.8088	—	—
	P_{E_f}	0.1596	0.1268	—	—
	P_{μ_z}	0.1077	0.0963	—	—
	P_{μ_x}	0.0290	0.0242	—	—
	P_{μ_y}	0.0011	0.0014	—	—

can see that for the resonant systems with thin ferrite rods (Fig. 3(a) and (b)), situated centrally in the dielectric resonator, the resonant frequency shift is predominantly connected with the change in the parallel component μ_z' of the permeability tensor (for these systems, p_{μ} and $p_{\kappa} \ll p_{\mu_z}$). This is clear since for the H_{018} -mode resonators the H_z component of the magnetic field has its maximum value at the dielectric resonator axis, while the remaining field components (H_{ρ} , E_{ϕ}) have minima there. For the resonant systems with ferrite disks, the influence of particular tensor components on the resonant frequencies is different for different disk positions. For the resonant system shown in Fig. 3(d) the parallel μ_z' component has a dominant influence on the resonant frequency values but that of μ' is also significant. For the resonant system shown in Fig. 3(c) the influence of the μ' component is dominant but the influences of κ' and μ_z' are nevertheless significant. The maximum tuning bandwidth which can be obtained for a particular resonant system and ferrite material depends on the

TABLE II
MAXIMUM TUNING BANDWIDTHS AND Q_f FACTORS OF THE
RESONANT SYSTEMS SHOWN IN FIG. 3

Case in Fig. 3	$\Delta f_{\max}/f_r$ [%]	$Q_{f\mu}$	$Q_{f\epsilon}$	Q_f
a	4.13	4290	20020	3530
b	4.36	4018	20390	3356
c	1.99	12200	27900	8490
d	1.82	8300	5260	3220

$$\mu' = \mu'_z = 0.7; \kappa' = 0.
\tan \delta_{\epsilon_f} = 15 \times 10^{-4}; \tan \delta_{\mu_d} = 10^{-3}.$$

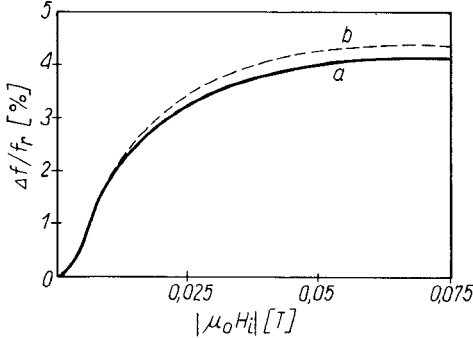


Fig. 5. Theoretical normalized tuning characteristics for the resonant quasi- $H_{01\delta}$ -mode systems shown in Fig. 3(a) (solid line) and in Fig. 3(b) (dotted line). Dependence of permeability tensor components on the dc magnetic field is taken to be the same as in Fig. 2.

admissible Q factor degradation of the resonant system due to ferrite losses. Let us define the Q_f factor depending on ferrite losses only as follows:

$$Q_f^{-1} = Q_{f\epsilon}^{-1} + Q_{f\mu}^{-1} \quad (4)$$

where

$$Q_{f\epsilon}^{-1} = p_{\epsilon} \epsilon_f'' / \epsilon_f'$$

and

$$Q_{f\mu}^{-1} = p_{\mu} \mu'' / \mu' + p_{\kappa} \kappa'' / \kappa' + p_{\mu_z} \mu_z'' / \mu_z'.$$

For a ferrite in the completely demagnetized state $\kappa'' = 0$ and $\mu_z'' = \mu'' = \mu_d' \tan \delta_{\mu_d}$, where μ_d' and $\tan \delta_{\mu_d}$ denote the scalar permeability and the magnetic loss tangent of demagnetized ferrite respectively. Also $\epsilon_f'' = \epsilon_f' \tan \delta_{\epsilon_f}$, where ϵ_f' and $\tan \delta_{\epsilon_f}$ denote the permittivity and the electric loss tangent of the ferrite. Table II gives results of computations of the maximum tuning bandwidths and Q_f factors (for demagnetized ferrites only), and Figs. 5 and 6 present tuning characteristics of the resonant systems shown in Fig. 3. It was assumed that the real parts of the ferrite permeability tensor components change as in Fig. 2 and that $\tan \delta_{\epsilon_f} = 15 \times 10^{-4}$ and $\tan \delta_{\mu_d} = 10^{-3}$ (these values were given by the manufacturer of the lithium-titanium ferrite). One can see that for the resonant systems shown in Fig. 3(a)–(c) the product of the tuning bandwidth and the Q_f factor is approximately the same, while for the resonant system shown in Fig. 3(d) it is about three times smaller. The reason for this is that in the last case dielectric losses constitute a large part of the total losses in ferrite medium, since the ferrite disk is situated at the

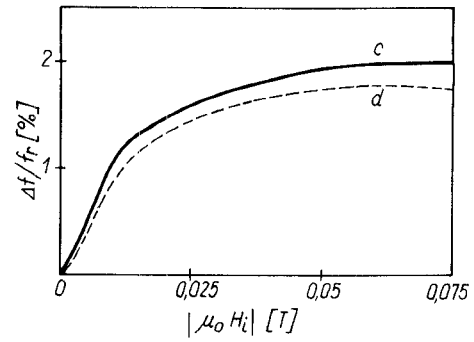


Fig. 6. Theoretical normalized tuning characteristics for the resonant quasi- $H_{01\delta}$ -mode systems shown in Fig. 3(c) (solid line) and in Fig. 3(d) (dotted line). Dependence of permeability tensor components on the dc magnetic field is taken to be the same as in Fig. 2.

position where the electric field has significant magnitude. Hence such ferrite position is not recommended for tuning of the resonant system. The remaining systems are equivalent from this point of view. However they are not equivalent if one considers efficiency of tuning defined as

$$\eta = \Delta f Q_f / H_{\text{ext}} \quad (5)$$

where Δf is the tuning bandwidth, and H_{ext} is the external magnetic dc field required for obtaining such a bandwidth.

For a thin ferrite rod the internal magnetic dc field in it $H_i \approx H_{\text{ext}}$ since the demagnetizing factor $N_z \approx 0$. For a flat ferrite disk $N_z \approx 1$ so $H_i \approx H_{\text{ext}} / \mu_{\text{dc}}$, where μ_{dc} denotes the relative static permeability of the ferrite. Hence the resonant system with a ferrite in the form of a thin rod makes it possible to obtain higher tuning efficiency. It will be shown in the next part of this paper that it is possible to build a close dc magnetic circuit which includes a microwave ferrite rod in order to obtain an extremely high tuning efficiency. We shall remark that it is also possible to reduce demagnetizing effects for resonant systems with ferrite disks by applying the dc magnetic field perpendicularly to the z axis, as was done in [4]. In this case, however, the electromagnet system must have a larger air gap since the transverse dimensions of the resonant system shield are usually larger than those along the z axis. Since in this case the electromagnetic field distribution in the resonant system loses the axial symmetry, it was not possible to analyze this case carefully with the aid of our computer program.

III. EXPERIMENTS

In our experiments the ring dielectric resonator used is made of dielectric with $\epsilon' = 14.95$ and $\tan \delta_{\epsilon} = 1.6 \times 10^{-4}$ (made by Polfer of material with the catalog symbol HF-D-15). The resonator has an external diameter of 8.00 mm, an internal diameter of 3.50 mm, and a height of 8.00 mm. Lithium-titanium ferrite rods (2.96 mm in diameter) with $\epsilon_f' = 16.2$, $\tan \delta_{\epsilon_f} \leq 15 \times 10^{-4}$, and $\tan \delta_{\mu_d} \leq 10^{-3}$ were used for tuning (made by Polfer of material with catalog symbol HF-L-260). The resonant frequencies and the unloaded Q factors of the resonant systems were measured using a swept-frequency Q factor meter made at the Institute of

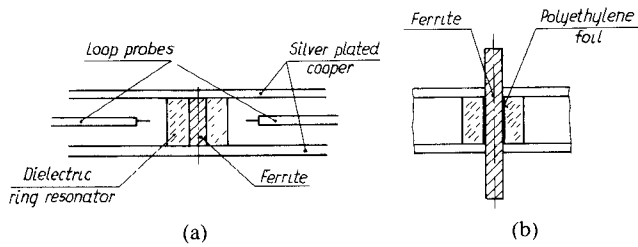


Fig. 7. Parallel-plate quasi- H_{011} -mode resonant configurations used in experiments. (a) The system with short ferrite rod ($h_f = h$). (b) The system with long ferrite rod ($h_f = 25$ mm).

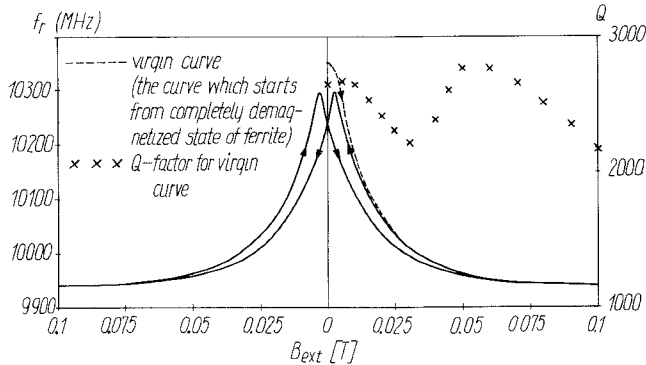


Fig. 8. Measured quasi- H_{011} -mode resonant frequencies and unloaded Q factors versus external dc magnetic field intensity for the resonant system shown in Fig. 7(b). $h = L = 8.00$ mm, $a = 4.00$ mm, internal diameter of the ring resonator $d_i = 3.50$ mm, $r_f = 1.48$ mm, $h_f = 25$ mm, $\epsilon'_f = 16.2$, $\epsilon' = 14.95$ (virgin curve is marked with dotted line).

Microelectronics and Optoelectronics, Warsaw University of Technology. The first group of experiments was performed using a large electromagnet system with a 50 mm air gap between electromagnet poles having a 250 mm diameter. The maximum magnetic induction to be obtained in the air gap was 0.7 T. In this first group of experiments, two parallel-plate dielectric resonator systems, sketched in Fig. 7, were used with coaxial cables terminated by small loops used for coupling.

The results of measurements of the quasi- H_{011} -mode resonant frequencies and unloaded Q factors for these systems, without and with ferrite rods, in the completely demagnetized state and with 0.7 T dc magnetic field applied, are presented in Table III. It should be pointed out that high magnetic field intensities, exceeding 0.6 T, are required to reduce the magnetic losses to a very small value. One can observe that experimental results (Table III) agree quite well with theoretical ones (Table I). Small differences between them are mainly due to differences between the actual parameters of the resonant system and those assumed for the theoretical computations. For example, the internal dimension of the ring resonator was assumed to be 3.00 mm in the theoretical computations but it was 3.50 mm in the experiments. Since no essential differences in the Q factor values and tuning characteristics between systems with short and with long (25 mm) ferrite rods were observed and the system with the long ferrite exhibits slightly larger tuning bandwidth, further results will be presented for this system only. Fig. 8 gives

TABLE III
MEASURED VALUES OF THE QUASI- H_{011} -MODE RESONANT FREQUENCIES AND UNLOADED Q FACTORS FOR THE PARALLEL-PLATE RESONANT SYSTEMS SHOWN IN FIG. 7

External magnetic DC field applied	Resonator filling	f_r (MHz)	Q
Any	Without ferrite	10128	4570
Completely demagnetized ferrites with no field applied	Short ferrite rod $h_f = L = 8.00$ mm	10332	2670
	Long ferrite rod $h_f = 25$ mm	10348	2655
0.7 T magnetic field applied	Short ferrite rod $h_f = L = 8.00$ mm	9906	4330
	Long ferrite rod $h_f = 25$ mm	9914	4270

$h = L = 8.00$ mm; internal diameter of the ring resonator $d_i = 3.50$ mm; $r_f = 1.48$ mm; $\epsilon'_f = 16.2$; $\epsilon' = 14.95$; $\tan \delta_e = 1.6 \times 10^{-4}$.

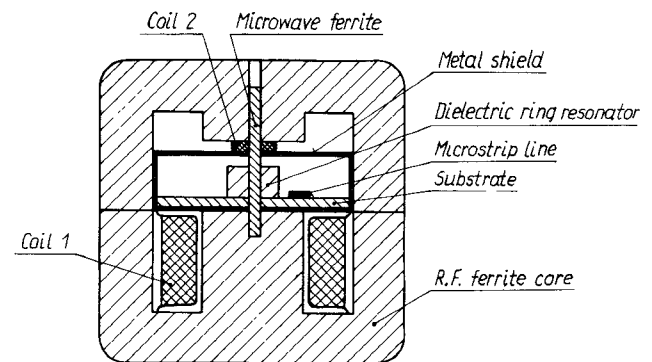


Fig. 9. Shielded quasi- H_{018} -mode magnetically tuned resonant system with dc magnetic field circuit which allows very efficient magnetic tuning.

results of the quasi- H_{011} -mode resonant frequency and the unloaded Q factor measurements versus dc external magnetic induction values for the resonant system shown in Fig. 7(b). One can observe that the tuning characteristics exhibit hysteresis effects. The maximum tuning range, defined as the difference between the resonant frequency value with ferrite in the completely demagnetized state and its value with 0.075 T dc magnetic field applied, is equal to 405 MHz (≈ 4 percent) and the minimum Q factor of the resonant system is higher than 2000 in the whole tuning range.

The results of experiments presented here confirm the validity of the theory presented in the first part of this paper. It has been proved that the tuning efficiency for the resonant system presented here is higher than that for the systems investigated in [3] and [4]. However, as mentioned earlier, the main advantage of this system is the possibility of constructing a close dc magnetic field circuit that includes the microwave ferrite as a part. An example of such a circuit is shown in Fig. 9. A very similar magnetic circuit was constructed using a typical core made of RF ferrite. The main arm of the core was cut and the holes were drilled appropriately by the manufacturer (Polfer). For the sake of comparison we used the same parallel-plate dielectric resonator used in the first part of our experiments (Fig. 7(b)) instead of the microstrip configuration depicted

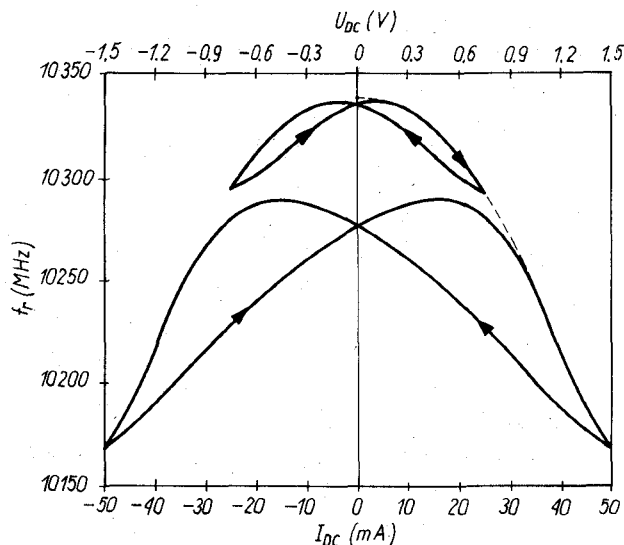


Fig. 10. Measured static tuning characteristics of the quasi- H_{011} -mode resonant system shown in Fig. 7(b) with the dc magnetic field circuit as in Fig. 9. Parameters of the resonant system are the same as in Fig. 8 (virgin curve is marked with dotted line).

in Fig. 9. The width of the silver-plated copper plates short-circuiting the dielectric resonator was limited to 38 mm. By the internal dimensions of the core, however, as one would expect, we did not observe the Q factor degradation of the resonant system due to radiation. The results of the experiments are presented in Fig. 10. It is seen that a tuning power of only 75 mW is required to obtain a 120 MHz tuning bandwidth. Despite the hysteresis effects the tuning characteristics are almost linear for wide ranges of tuning current values. Each of the two curves shown in Fig. 10 was found independently beginning with the completely demagnetized state of the ferrite. It is not easy to reach the completely demagnetized state of the ferrite with a direct current source. If instead of the direct current source a 50 Hz alternating current source was applied, it was always easy to reach this state by gradually decreasing the magnitude of the tuning current. We expect, in further applications of this system of tuning, to obtain high tuning speeds since a ferrite material will be used as the core.

IV. CONCLUSIONS

As presented in this paper, the rigorous theory of magnetic tuning of the H_{011} -mode dielectric resonators allows the design of various ferrite-tuned resonant systems having axial symmetry, with a very high accuracy. It has been proved that the most effective tuning of such resonant systems can be accomplished by inserting a thin ferrite rod

through the hole in a cylindrical ring resonator and its shield. The ferrite material used for tuning should possess small magnetic losses and a small μ'_d value. To get the lowest consumption of tuning power, a close dc magnetic field circuit should be built including the microwave ferrite as a part. Applications of such systems are expected to include the construction of very high Q , electronically controlled dielectric resonator oscillators.

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Since 1973 he has been associated with the Institute of Microelectronics and Optoelectronics, Warsaw University of Technology, where he is now an Assistant Professor. While on leave during the academic year 1981/1982 he held the position of Lecturer I in the Physics Department, University of Jos, Nigeria. His current research interests are concerned with numerical methods in electromagnetic field theory and the measurement of dielectric and magnetic material properties at microwave frequencies.